EXPERIMENTAL VERIFICATION OF STABILITY CHARACTERISTICS FOR THERMAL ACOUSTIC OSCILLATIONS IN A LIQUID HELIUM SYSTEM

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INTRODUCTION

Thermal acoustic oscillations (TAOs) are frequently observed when a tube is inserted into a cryogenic system, particularly when filled with liquid helium. A step temperature profile along the length of the inserted tube has typically been assumed by previous researchers in the development of a theoretical model for such oscillations[3,4]. Such theoretical predictions, however, have shown relatively large errors when compared with the experimental verifications. In this study, thermal acoustic oscillation parameters (including oscillation pressure amplitude and frequency) and temperature profile along the length of the oscillation tube have been measured simultaneously to investigate the effect of temperature profile on the characteristics of TAOs. Relatively steep temperature profiles have been observed. In addition, stability characteristics for thermal acoustic oscillations in a liquid helium system with a continuous temperature profile along the length of the tube have been predicted as well as the oscillation effect of the length ratio between the warm and cold sections of the tube. The latter effect has been observed experimentally to be very sensitive to the stability characteristics of TAOs. Good agreement has been achieved between experimental results and theoretical predictions.

THEORY

Thermal acoustic oscillations are initiated by the large temperature gradient existing along the length of the tube inserted into the dewar. During the pressure oscillations, the gas inside the tube will oscillate as well. Accompanying this oscillations is a viscous resistance that is proportional to the velocity gradient near the tube wall and the viscosity of the fluid. Both the amplitude and direction of the fluid velocity change during the oscillation period indicating the existence of a significant inertial force. It appears that the heat transfer to the fluid in the tube due to the large temperature difference between the warm and cold ends is the driving force for these thermal acoustic oscillations[1,5].

In general, hydrodynamic equations (that is, equations of continuity, momentum, energy, and state) can be used to describe and solve this oscillation problem. However, the

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complexity of these equations makes it impossible to solve them analytically. On the other hand, the velocity, pressure, density and temperature of the fluid in the tube oscillate around their individual mean values during thermal acoustic oscillations. This implies that these parameters can be perturbed around their mean values and the general hydrodynamic equations can be greatly simplified when only the stability characteristics for TAOs are required[2,3]. In an early study Rott[3] made several assumptions during the perturbation of these general equations, including (1) negligible radial variations of pressure and mean temperature; (2) time variation of the acoustic variable expressed as $e^{i\omega t}$; and (3) tube radius r much smaller than the tube length L. The resulting equation involving the acoustic pressure p_l is

$$[1+(\gamma-1)f^*]p_1+\frac{d}{dx}\left[\frac{a^2}{\omega^2}(1-f)\frac{dp_1}{dx}\right]+\frac{a^2}{\omega^2}\frac{(f-f^*)}{(1-\mathrm{Pr})}\theta\frac{dp_1}{dx}=0 \qquad (1)$$

where γ is the heat capacity ratio, x the tube length parameter, a the local sonic speed, ω the oscillation frequency, Pr the Prandtl number, and

$$f=\frac{2I_1(\eta_0)}{\eta_0 I_0(\eta_0)}; \qquad f^*=f(\eta_0\sqrt{\mathrm{Pr}}) \qquad (2)$$

and

$$\theta=\frac{1}{T_m}\frac{dT_m}{dx} \qquad (3)$$

Here, T_m is the temperature of the tube wall, I_0 and I_1 are the Bessel functions of the first kind with an order of zero and one, respectively, and η_0 is defined as

$$\eta_0=r\sqrt{\frac{i\omega}{\nu}} \qquad (4)$$

where ν represents the kinematic viscosity of the fluid in the tube and r the radius of the oscillation tube.

It should be noted that the oscillation frequency ω is generally a complex number ($\omega=\omega_r+i\omega_i$), its real part ω_r represents the actual oscillation frequency during TAOs while its imaginary part ω_i represents the amplification of the oscillation. Therefore, a negative value of ω_i will result in an exponential amplification of the pressure amplitude with time during thermal acoustic oscillations; thus, TAOs will be initiated and sustained when $\omega_i<0$. On the other hand, any disturbance in a thermal acoustic oscillation system can be damped if the value of ω_i is positive. Therefore, the stability characteristics for TAOs can be determined from the characteristics of the imaginary part of the oscillation frequency. It is apparent that the marginal boundary separating the stable and unstable regions for TAOs occurs when the value of ω_i is equal to zero.

EXPERIMENTAL SYSTEM AND RESULTS

An experimental system has been established to investigate the characteristics for thermal acoustic oscillations in a liquid helium system as shown in Fig. 1. The test tube is inserted into an open mouth dewar during the experiment. TAOs are observed in this tube under appropriate test conditions. A pressure sensor is mounted at the warm end of the tube to measure the oscillation pressure in the tube. This is where the maximum pressure amplitude can be obtained. The pressure observed at the open end of the tube, however, is essentially equal to that in the dewar; thus, a much smaller oscillation pressure amplitude

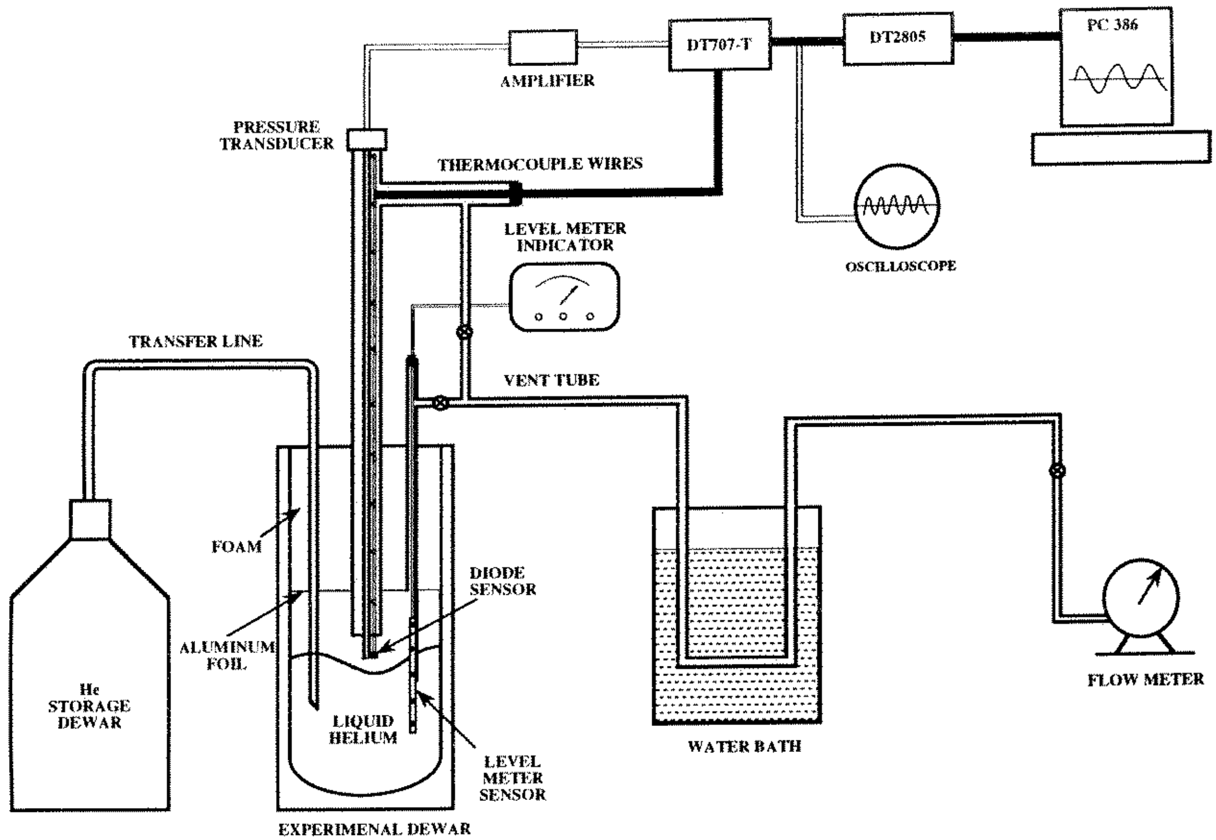

Fig.1. Schematic of the thermal acoustic oscillation experimental system.

occurs at this location since the surroundings in the dewar greatly reduces the impact of these oscillating pressures.

In order to determine the effect of tube radius and length on the stability characteristics of TAOs, test tubes with different radii and lengths were used during the experimental run. A double tube design was adopted to more rapidly elevate or lower tubes of different radii into the liquid helium dewar. This permitted installation of thermocouples on the outside of the test tube wall while maintaining leak tightness of the system. A temperature profile along the length of the test tube has been measured to investigate the effect of temperature profile on the stability characteristics associated with TAOs. Typical experimental results for the temperature profile along the length of the oscillation tube along with related pressure waves in the tube are shown in Fig. 2, where ξ is the ratio of the tube length in the warm section of the tube to that in the cold section of the tube. It can be observed that a relatively steep temperature gradient exists along the tube, while the resulting pressure wave is approximately a sine wave during thermal acoustic oscillations.

STABILITY RESULTS

The stability characteristics for thermal acoustic oscillations can be obtained by numerically solving Eq. (1). A step-temperature profile along the length of the tube (i.e., assuming a constant temperature T_h for the warm section of the tube and T_c for the cold section of the tube) has been used by Rott to determine the stability curves for TAOs. However, experimental results show that a continuous temperature profile exists along the oscillation tube.

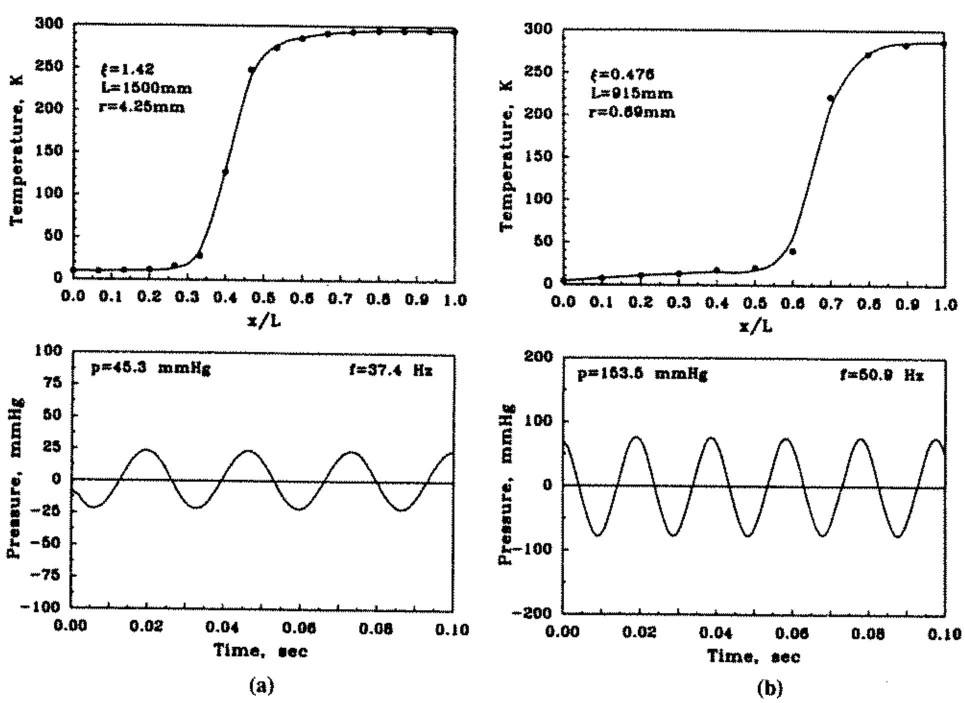

Fig. 2. Temperature profile and related oscillation pressure wave for TAOs in helium system. (a) L=1.5m, r=4.25mm, ξ=1.42; (b) L=0.915m, r=0.69mm, ξ=0.476.

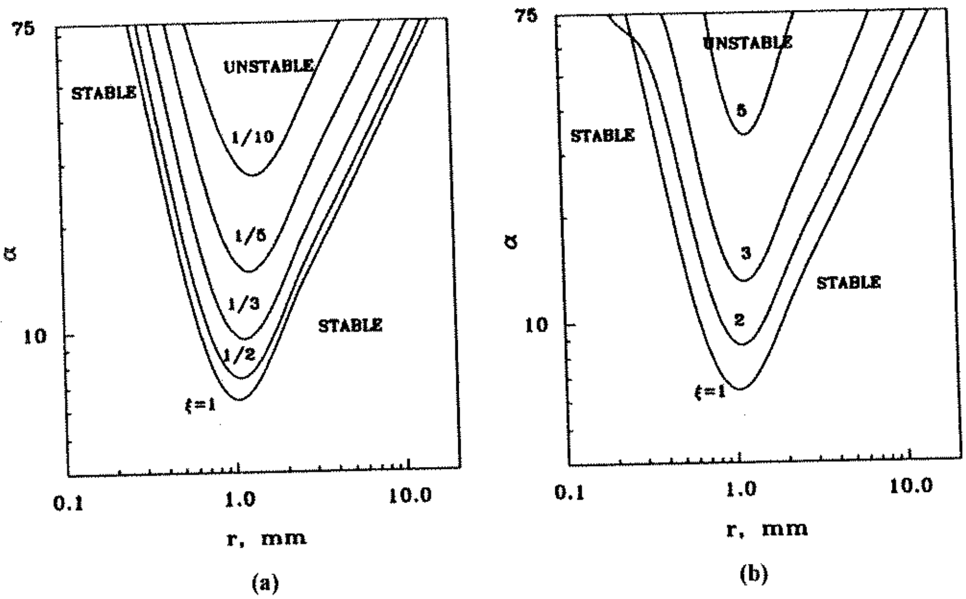

Fig. 3. Stability curves for TAOs in a helium system with a continuous temperature profile along the length of the tube. (a) $\xi \leq 1$; (b) $\xi \geq 1$.

By using a continuous temperature profile close to that experimentally obtained along the test tube, stability curves for thermal acoustic oscillations in a helium system have been obtained. The results are shown in Fig. 3, where α represents the ratio of the temperature at the warm end of the tube to that at the cold end of the tube (T_H/T_c), while ξ is the ratio of the tube length in the warm section of the tube to that in the cold section of the tube (L_H/L_c). The location separating the warm and cold sections of the tube is defined as the point where the temperature is equal to $(T_h+T_c)/2$. In plotting this figure, the temperature at the warm end of the tube T_h is assumed to be constant since it is approximately equal to the ambient temperature when the tube is inserted into a cryogenic dewar. However, the temperature at the cold open end of the tube can vary not only because of a temperature gradient that exists in the ullage space of the dewar but also because axial heat conduction occurs along the length of the tube. A tube length of one meter is used in numerically calculating these stability curves.

Each stability curve consists of left and right branches which enclose an unstable region. In addition, a minimum temperature ratio is required for initiating TAOs for each stability curve. The largest unstable region for initiating TAOs occurs when the value of the length ratio ξ is equal to unity. These results are qualitatively the same as those predicted by Rott with an assumption of a step-temperature profile along the length of the tube.

A thermal acoustic oscillation system includes driving, viscous and inertial forces due to the large temperature gradient along the length of the tube and the oscillating flow of the gas in the tube. When the tube radius is relatively small, the viscous resistance is dominant and thermal acoustic oscillations may be damped by reducing the radius of the tube; this establishes the rationale for the existence of a left branch associated with the stability curve. However, more mass will oscillate in the tube when the tube radius becomes relatively large which results in a larger inertial damping. This establishes the rationale for the existence of a right branch for the stability curve.

Note that the viscous force in the warm section of the tube is usually much larger than that in the cold section of the tube since the viscosity of the fluid at the higher temperature is much larger than that at the lower temperature. However, the density of the fluid at the lower temperature is greater than that at the higher temperature. In addition, the driving force will also be changed with a change in the length ratio ξ because the tube surface exposed to the warm environment has been changed. Therefore, a change in the length ratio between the warm and cold sections will change the relative relationship between the driving, viscous and inertial forces and results in different stability characteristics associated with these thermal acoustic oscillations.

The most significant difference between the stability characteristics for TAOs with a continuous temperature profile and a step-temperature profile along the length of the tube is the unsymmetrical oscillation characteristics between $\xi<1$ and $\xi>1$; i.e., the unstable region for the stability curve when $\xi>1$ is much less than when $\xi<1$. For example, the minimum temperature ratio required for initiating TAOs when $\xi=5$ is about 33, while it is only about 9.5 when $\xi=1/5$. This indicates that thermal acoustic oscillations can be more easily damped by increasing the length ratio ξ when $\xi>1$ than by decreasing the ξ when $\xi<1$.

The overlap of stability curves in Fig. 3b is attributed to the phase shift that occurs between the driving force and the viscous resistance since both of these forces periodically change with time[1]. An oscillation can still be initiated when the amplitude of the resistance is larger than the amplitude of the driving force if a proper phase shift exists between these two forces. However, this occurs only when the tube radius is very small and the length ratio ξ is greater than unity.

EXPERIMENTAL VERIFICATIONS

Stability characteristics for thermal acoustic oscillations in a liquid helium system using a continuous temperature profile along the length of the tube are verified in Fig. 4 with different values of the length ratio between the warm and cold sections of the tube. As the numerical stability results were obtained with a tube length of one meter which may be different from that used during the experimental run, the actual tube radius was corrected before being plotted in these figures. The values of the corrected tube radius r' for tube length L' is calculated from

$$r' = \frac{r}{\sqrt{L}}\sqrt{L'} \tag{5}$$

where r is the actual tube radius, L the actual tube length and L' the tube length used in calculating the stability curves (L' is equal to one meter in this study).

In addition, the piezoresistive pressure sensor mounted at the warm end of the tube introduces an additional volume (0.4 cm^3) to the thermal acoustic oscillation system. The effect of this additional volume on the stability characteristics associated with TAOs is essentially negligible when thermal acoustic oscillations occur in a tube with a relatively large tube radius. However, the effect of this volume can not be neglected when the tube radius is relatively small since this additional volume may be even larger than the volume associated with just the test tube. The equivalent length ratio ξ' (which is always larger than the actual tube length ratio) can be calculated from

$$\xi' = \frac{L_h + L_{hs}}{L_c} \tag{6}$$

where L_h and L_c are the actual lengths of the tube in the warm and cold sections, respectively, and L_{hs} is the equivalent length in the warm section of the tube due to the additional volume introduced by the pressure sensor. This quantity can be determined by

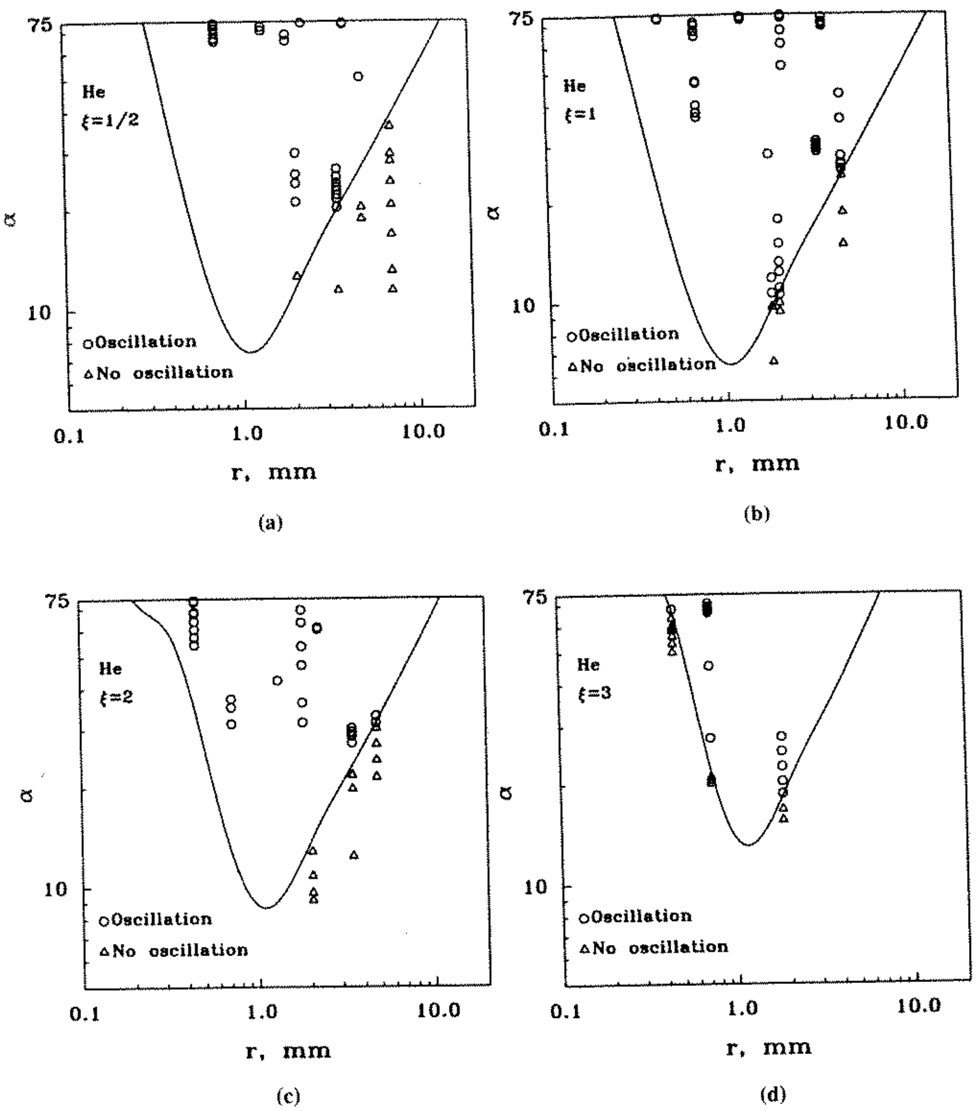

Fig. 4. Experimental verification of the stability curves.
(a) ξ=1/2; (b) ξ=1; (c) ξ=2; (d) ξ=3.

$$L_{hs} = \frac{V_s}{\pi r^2} \quad (7)$$

where V_s is the additional volume of the pressure sensor and r is the radius of the test tube.

These figures indicate that the agreement between the experimental results and the theoretical predictions is very good after correcting for the length ratio ξ and the tube radius r. Note that the volume correction can increase the value of the tube length ratio significantly when the tube radius is very small. This suggests that adding a relatively large volume at the warm end of the tube can serve as an effective method for damping thermal acoustic oscillations when the tube radius is small and the length ratio ξ is greater than unity.

CONCLUSIONS

The stability characteristics for thermal acoustic oscillations in a helium system have been investigated in this study. Conclusions drawn from these results indicate that

1. The stable and unstable regions for TAOs can be adequately determined if a continuous temperature profile along the length of the tube is used.
2. Each stability curve is represented by two distinct branches when plotting α versus increasing radius of a tube closed at the warm end and inserted into a cryogenic system. The left branch is dominated by a viscous resistance while the right branch is dominated by an inertial force.
3. A tube length ratio of one results in a maximum unstable region for TAOs. Either an increase or a decrease in the length ratio ξ results in a smaller unstable region. A relatively symmetrical characteristic in the stability curve was observed when a step-temperature profile was assumed along the length of the tube. However, a much smaller unstable region was predicted and experimentally verified when a continuous temperature profile was used along the length of the tube when $\xi>1$ than was observed when $\xi<1$.

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